EFFECT OF OPENINGS ON THE BEHAVIOR OF R.C. FLAT PLATES PART (II) : PLATES WITH EDGE BEAMS

By

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ABSTRACT

Openings in flat plate systems are essential for stairways, service ducts,...etc. When openings exist in a plate, even if the sizes of the openings are relatively small as compared with the size of the plate itself, they are supposed to affect the structural performance of the plate and to cause reduction in its strength, stiffness and deformation capability, depending on the locations and number of the openings, The aim of this paper is to study the effect of the location of these openings on the behavior of R.C. plates with edge beams. This requires proper combination of both the column, beam, and plate elements. A high precision isoparametric triangular thin layered element is used for modelling the studied plates while beams are modelled by a specially developed thick layered one, which takes shear deformation into consideration. Columns are represented by a group of springs of variable stiffnesses. Based on the comparison between the numerical results and those obtained by the available methods stated in the Egyptian and American Standards, recommendations for the analysis and design of reinforced concrete flat plates with openings are proposed.

1. INTRODUCTION

In flat plates with marginal (edge) beams, the slab column connection is a complex region since it is subjected to high shear stresses produced by the combination of axial load and bending moments. Introducing openings in plates and walls affects the behavior of these plates [1,2,3]. The effect of opening location, dimensions and aspect ratio on the behavior of flat plates without edge beams was studied earlier by the authors in the first part of this series [4]. However, the research in the effect of openings on the plate column connection is limited especially if the opening is very close to the column. Since at the column-plate and beam-plate junction, the behavior of the column, the beam and the plate is complex, the effect of the interface between these regions would be accurately evaluated through three dimensional (3-D) analysis [5]. However, this would involve the difficulties related to such space analysis and would loose the merits of two-dimensional analysis using the layered approach. A high precision isoparametric triangular thin layered element model was applied successfully to the first part of this series [4] but it needs further development to model the column-plate and beam-plate connections accurately.

The object of this paper is to study the effect of openings on the behavior of plates with edge beams. Beams are modelled by a specially developed thick layered triangular element which takes shear deformation into consideration while columns are represented by a group of springs. In addition, the effect of reinforcement detailing around the openings on the behavior of such plates is taken into consideration. Comparisons are made between the numerical results obtained by the proposed computer program and the equivalent frame method of the ESS-95 Code [6] and the equivalent frame method stated in the ACI-95 Code [7].

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2. FINITE ELEMENT ANALYSIS

The layered finite element model developed earlier by the first author [8], and modified to include the concrete and steel layers in the first part of this investigation [4] was further developed to include a better simulation to plate-column connection and a layered beam element for edge beams. It is worth mentioning that the orientation of the reinforcement in the steel layer is included in the computer program.

2.1 Plate Element Modelling

The layered triangular thin element was used in modelling the studied plates. This element is considered to be a high precision one since it has high degrees of compatibility per node. The nodal parameters are three displacements u,v, and w along the global Cartesian coordinates X,Y, and Z, two strains and shear strains u_x, v_y and u_y, v_x, two rotations w_x, w_y, two curvatures w_{xx}, w_{yy} and twist w_{xy}. These comprise twelve nodal degrees of freedom and thirty six degrees of freedom per element, (see Figure 1-a). The displacement field is defined as:

$$U = \Sigma N^{(3)}_i . u_i$$
$$V = \Sigma N^{(3)}_i . v_i \qquad (1)$$
$$W = \Sigma N^{(5)}_i . w_i$$

where the nodal displacement vectors are given by,

$$\{u_i\}^T = \{u_1, u_{x1}, u_{y1}, u_2, \ldots\ldots, u_{y3}\}$$
$$\{v_i\}^T = \{v_1, v_{x1}, v_{y1}, v_2, \ldots\ldots, v_{y3}\} \qquad (2)$$
$$\{w_i\}^T = \{w_1, w_{x1}, w_{y1}, w_{xx1}, w_{xy1}, w_{yy1}, w_2, \ldots\ldots, w_{yy3}\}$$

where;

$u_{x1} = (\partial u / \partial x)$ for node No. 1

$w_{xx1} = (\partial^2 w / \partial x^2)$ for node No. 1

etc.....

$N_i^{(3)}$ and $N_i^{(5)}$ are third order and fifth order shape functions at node i. It was demonstrated previously [8] that these high order shape functions achieve high precision of the finite element model.

The reinforced concrete element is considered as a group of concrete layers while the steel reinforcement is assumed as smeared steel layers, (see Figure 1-b). The full details of the formulation are stated elsewhere [4,8].

2.2 Modelling of A Column-Plate Connection

Mahmoud et al. [9] represented the column by a space frame system with 6 degrees of freedom (DOF) at each of its ends. This is considered as an accurate modelling but on the other hand it is a time consuming. The effect of columns on the slab at the slab-column interface falls within two limits [5]. A lower limit where the column is assumed to add no additional bending stiffness to the plate at their interface and an upper limit where the column makes the plate infinitely stiff in flexure within their interface. Practically, the plate rotational stiffness at the plate-column interface lies between the two extremes being much closer to the upper limit [10,11].

Among the several techniques for simulating the effect of columns on slab-column interface [5,12,13], the technique of representing the column by point supports with only axial stiffness located at the corners of the physical column [14] was considered to be the simplest one with a reasonable degree of accuracy. Therefore, the columns in this study, were represented by a group of springs at the nodes present within the column area with axial spring stiffness equal to the axial spring stiffness of the column [15], Figure (1-c). The change of stiffness of these springs and its effect on the behavior of plates was taken into consideration. The derivation of spring stiffnesses formulation is reported in details elsewhere [16].

2.3 Development of a Thick Layered Triangular Beam Element

In order to predict the edge beam behavior accurately in the analysis, it is vital to include the transverse shear deformation in the displacement based beam elements. This would need further development of the thin layered triangular element used previously [4,8] by adding shear deformation effects. Extra nodal degrees of freedom are generally utilized to represent shear deformations.

The first order shear deformation plate theory (SDPT) was described earlier [17,18]. To introduce the SDPT into the layered triangular element based on the classical plate theory, the transverse deflection is considered as a sum of two partial deflections, the deflection due to bending and the deflection due to transverse shear. Then, the displacement field in Equation (1) was modified as follows:

$$u(x,y,z) = u - z.w_x^{(b)}$$
$$v(x,y,z) = v - z.w_y^{(b)} \qquad (3)$$
$$w(x,y,z) = w^{(b)} + w^{(s)}(x,y)$$

where u, v and $w^{(b)}$ are the displacement fields for thin layered element defined earlier in Equation (1) and u(x,y,z), v(x,y,z) and w(x,y,z) are the in-plane and transverse displacement components at any point in the layered thick (beam) element along x,y and z axes respectively, (see Figure 1-d). Similarly, $w^{(s)}$ is the middle surface transverse displacement components due to bending and shear along the z axis.

In addition to the strains associated with the displacement field in Equation (1) which were derived elsewhere [4,8], the following shear strains were added to the formulation in order to include the shear deformation with the new displacement field in Equation (3);

$$\gamma_{xz} = w_x^{(s)}$$

$$\gamma_{yz} = w_y^{(s)} \tag{4}$$

where γ_{xz} and γ_{yz} are shear strains in planes x-z and y-z respectively.

$w_x^{(s)}$ and $w_y^{(s)}$ are the rotations around x and y axes due to shear effect.

The stress-strain relation for one layer of the laminated approach was given earlier [4] and the stress-strain relation for a layer due to transverse shear deformation is given by:

$$\begin{Bmatrix} \sigma_{xz} \\ \sigma_{yz} \end{Bmatrix} = \begin{bmatrix} Q_{44} & Q_{45} \\ Q_{45} & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{5}$$

where Q_{ij} are the transformed material constants [19].

The resultant forces and moments acting on a layer are obtained by integration of the stresses in each layer through the beam thickness. The famous force-deformation relationship [20], is written as:

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon_0 \\ k \end{bmatrix} \tag{6}$$

where $\{\varepsilon_0\}$ is the total extension and shear strains of the middle plane of the beam and $\{k\}$ is the vector of its bending and twisting curvatures. The matrices [A], [D] and [B] are the membrane, bending and coupling stiffness matrices.

The transverse shear force-strain relation becomes:

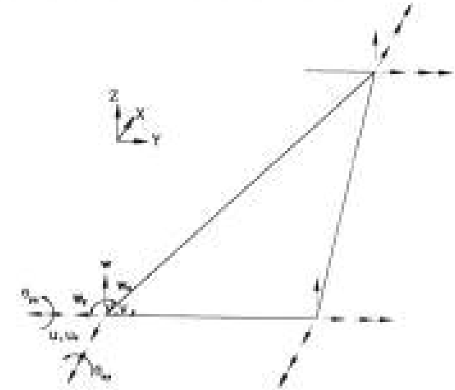
$$\begin{Bmatrix} Q_{xz} \\ Q_{yz} \end{Bmatrix} = \begin{bmatrix} S_{44} & S_{45} \\ S_{45} & S_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} \tag{7}$$

where S_{44}, S_{45} and S_{55} are the shear rigidities.

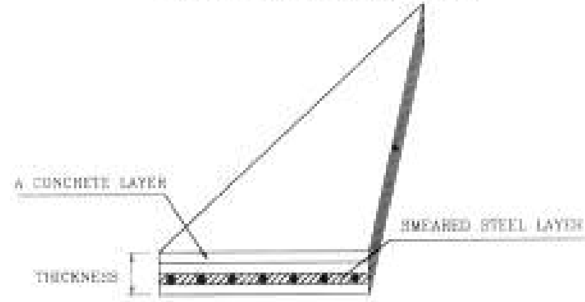
After adding the shear force-strain relation to the formulation, the finite element formulation is carried out easily and the stiffness matrix is derived as described elsewhere [8].

3. OUTLINE OF THE STUDIED PLATES

The model consisting of three by three plate panels and columns fixed at their far ends, which was used in the analysis of the first part of this series [4], was modified by adding edge beams. The panel dimensions were varied from 6.0m × 6.0m to 6.0m × 8.0m in order to assess the effect of plate aspect ratio. A finite element mesh for a part of one of the studied plates is shown in Figure (2). Three sizes of edge beam were considered in the analysis (25×45cm, 25×60cm and 25 ×90cm) in order to examine the effect of edge beam

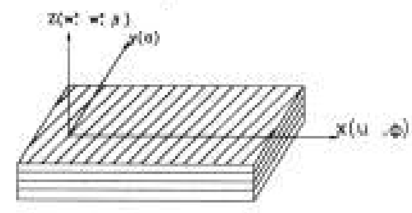


a- 36 DOF triangular finite element

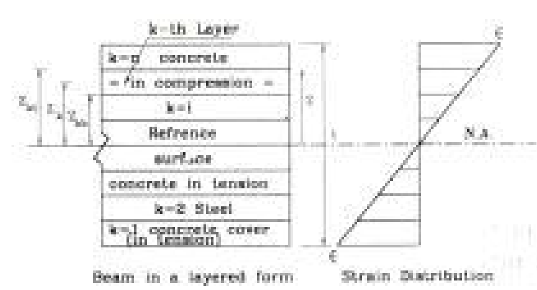


b- A layered system

c- Column- plate connection



Degrees of freedom and fiber angle of a thick layered beam element



A layered idealization of R.C. thick beam element

d- A thick layered beam element

Fig. 1- Finite element idealization of reinforced concrete plates

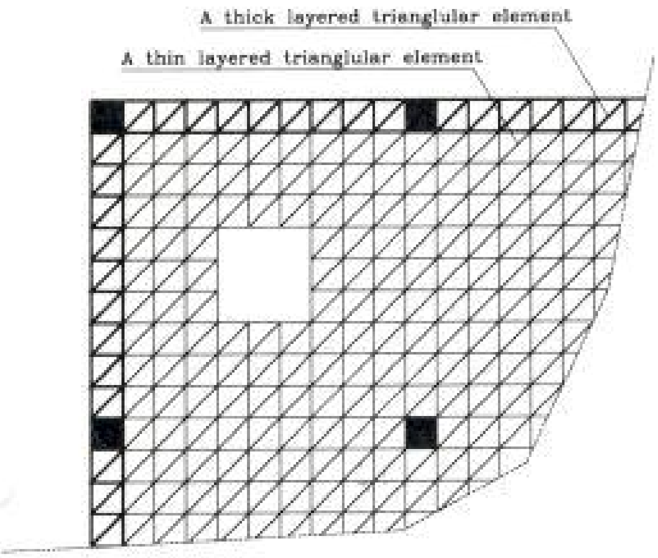


Fig. 2- A finite element model for a part of one of the studied plates

stiffness on the plate behavior. Since the opening size and aspect ratio were studied in the first part of this series, it was decided to maintain the opening dimensions 1.0m × 1.0m.

The loads, concrete characteristic strength, steel reinforcement rank and plate thickness were maintained as in the first part [4], 1400 kg/m^2 , 350 kg/cm^2 , 40/60 and 0.20 m, respectively.

The studied plate cases are divided into three main categories, i.e. S, C and F series, which refer to solid plate, plate with openings in the column strip and plate with openings in the field strip, respectively. These categories, in turn, are divided into subcategories based on the plate aspect ratio and the openings arrangement, (see Figure 3). The values 6, 7 and 8 which follow each series, as shown in Figure (3), refer to the panel dimension in the longitudinal direction. In addition to the arrangement of openings at the mid length of column strip and field strip panels (i.e. C-6, CE-6, CI-6, F-6,.... etc. as shown in Figure 3), new locations adjacent to the column and the edge beam were considered in this study (C-6*, CE-6*, CI-6*, FB-6).

4. RESULTS AND DISCUSSIONS

The effect of openings on the behavior of reinforced concrete plates with edge beams of variable size was investigated. The studied parameters include opening location, reinforcement detailing, modelling of plate-column connection and marginal beam depth. The resulting flexural moments for both column and field strips in both the longitudinal and the transverse directions were recorded and analyzed.

4.1 Effect of Local Arrangement of the Openings

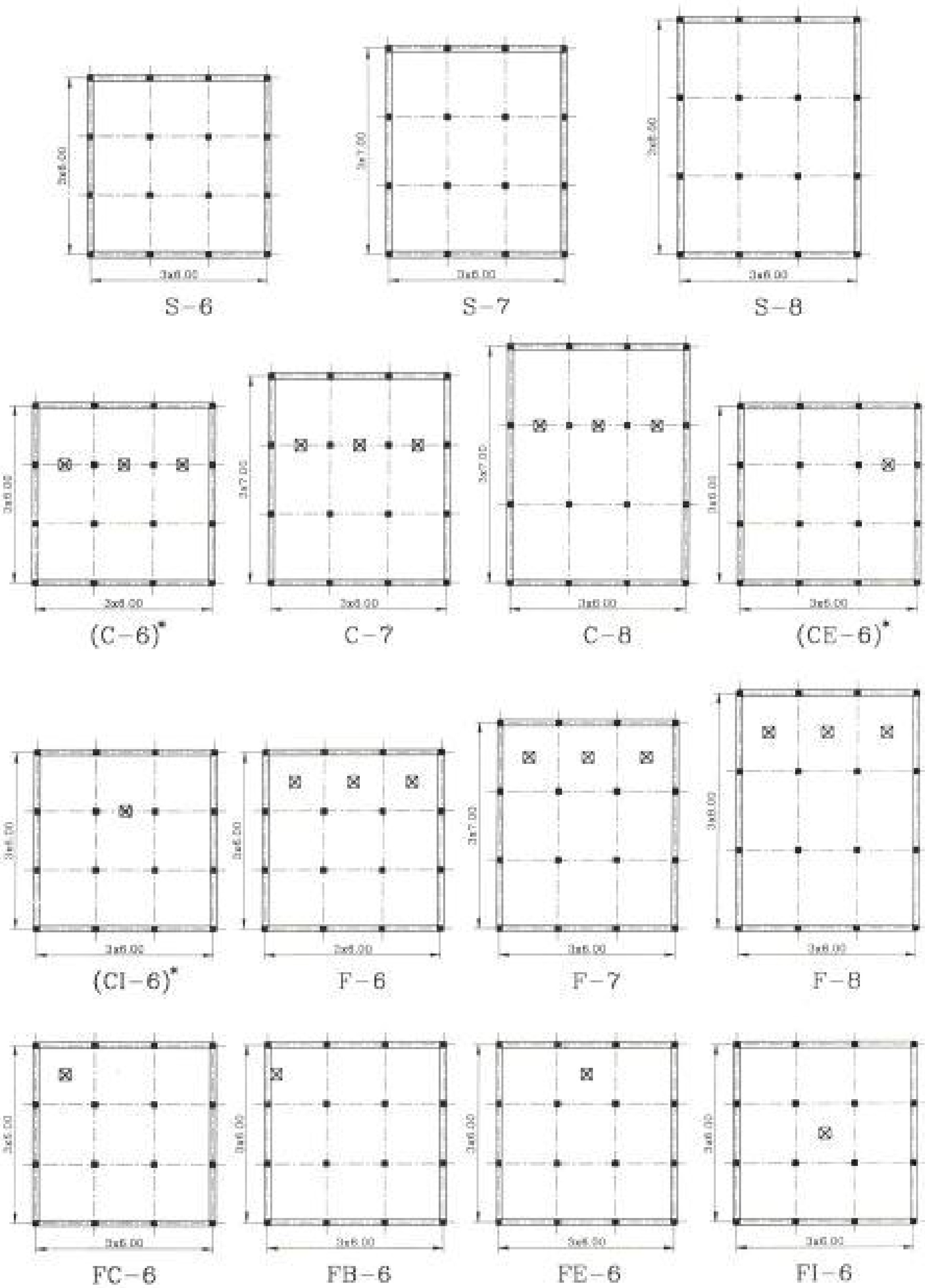
The effect of moving the opening from the mid-span between columns to be adjacent to the column is studied for plates with aspect ratio (L1/L2=1.0) and with edge beam 25×90cm. The results are shown in Figure (4) for the column strip flexural moments. It can be seen from the figure that locating the opening adjacent to the column edge leads to an increase in the overall flexural moments of the plate. However, the increase in the negative moment was very large compared to that of the positive moment. This may be attributed to the very complex state of stress from which the column-slab connection region suffers. It was noticed also, that the increase in the transverse moment was very high for the case of three openings in the same strip (C-6) compared with that for the case of one opening only (CE-6 or CI-6).

The behavior of the studied plates was monitored for various locations of openings, i.e. different distances from column edge, in order to find the most convenient position for the opening with negligible effect on the flexural moments in plates. It was found that a distance of about one tenth of the span away from the column face is reasonable. This finding almost agrees with the ESS-95 which prevents any opening in the area of the column head (0.25 the average span). It was found also that the amount of increase in the flexural moments was very high for openings in the intersection between two column strips with opening size greater than one eighth the strip width. This indicates that openings with that size are not recommended in such areas.

It was noticed that moving the opening location from the mid-span of the panel (FC-6) to be flushed to the edge beam (FB-6), see Figure 3, did not affect the field strip moments significantly.

4.2 Effect of Reinforcement Orientation around the Openings

In order to assess the capability of the computer program for modelling the steel reinforcement with anisotropic nature, diagonal reinforcement around the openings (see Figure 5-a) was considered in the analysis as an anisotropic smeared layer. The studied plate cases were the solid one S-6, the plate with openings in mid-length between columns C-6 and the plate with openings adjacent to columns edges C-6*. All plates were of panel dimensions 6.0m × 6.0m and with edge beams 25×90cm.



(*) Multiple cases for opening location (from midspan to column edge) are considered

Fig. 3- Arrangement of opening in different plate cases

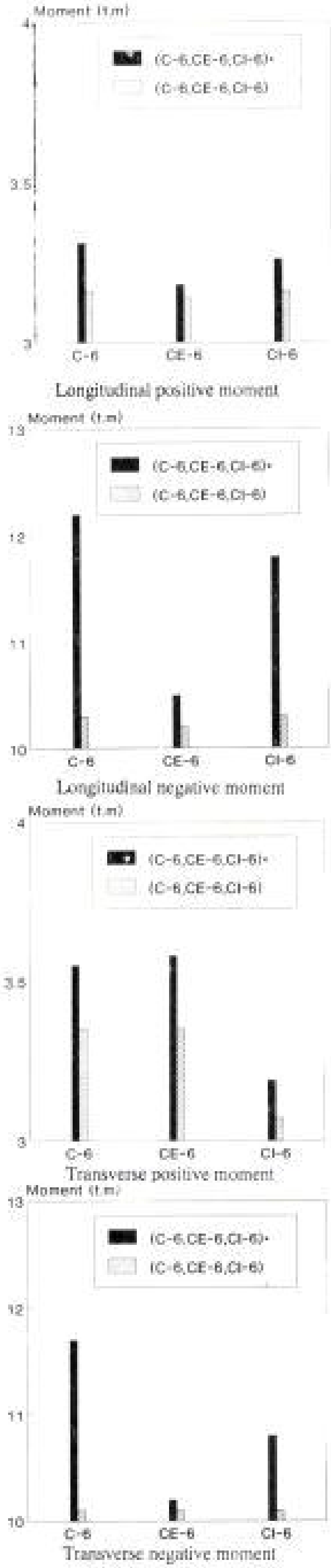
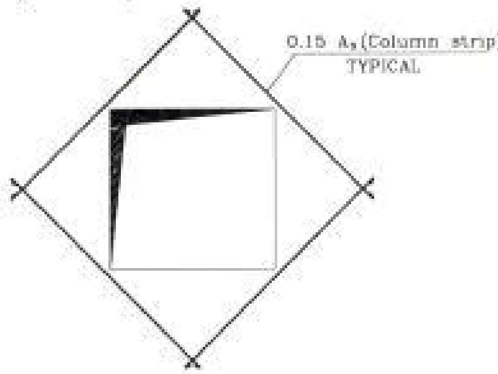


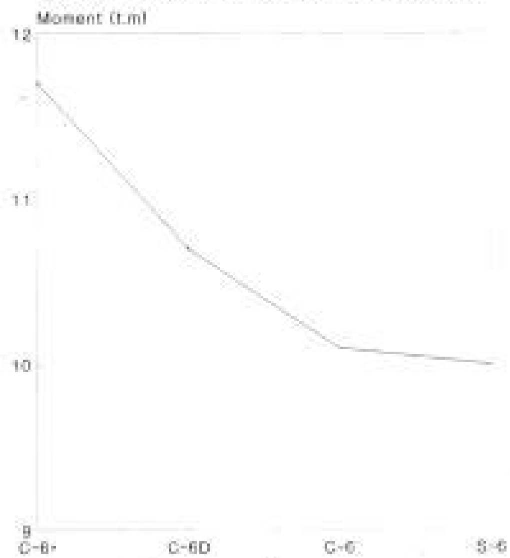
Fig. 4- Effect of openings arrangement on column strip moments

Figure (5-b) shows flexural negative moments in the transverse direction for the different studied cases. As it can be expected, the figure shows that the maximum moment value was for the case of openings adjacent to the columns and the minimum one was for the solid plate. It is noticed that the reduction of the moment by moving the openings from the location adjacent to the columns to be in the mid-length between the columns was about 14%. It is interesting to note that including an anisotropic layer of diagonal steel around the openings of about 0.15 from the main column strip steel leads to a reduction of the moment value by about 9%, i.e. it almost approaches the case of opening at mid-length of the plate.



C-6D

a- Additional diagonal reinforcement around opening



Transverse negative moment

b- Variation of moments with opening locations

Fig. 5- Effect of diagonal reinforcement around openings on the behavior of plates

4.3 Effect of Column-Plate Connection Rigidity

As was mentioned earlier in Section 2.2, the columns were modelled as a group of springs to assess the effect of column-plate connection rigidity on the resulting plate flexural moments. Figure (6) shows the column strip negative and positive moments for plate with three openings in column strip (C-6) and with edge beam 25 × 90. The results were obtained for stiffness values ranging from 0.025 to 100000 times EA/L in order to simulate the various rigidities of the column plate connection. It was found that the flexural moments were almost constant for stiffness greater than 1.0 times EA/L. For smaller stiffness values, the positive moments were noticed to be increased dramatically while the negative ones were decreased gradually.

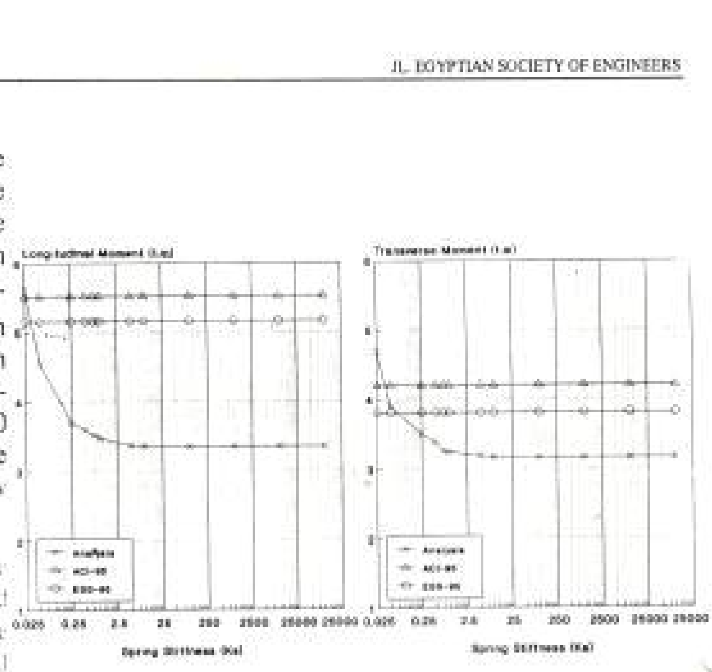
It can be seen from Figure (6) that the positive flexural moment values obtained by the methods stated in the codes of practice (ESS-95 and ACI-95) were higher than those obtained by the proposed analytical model, while the negative ones were lower than those obtained numerically. This could be attributed to the difference between methods of analysis used in the above mentioned codes and the present study since the codes of practice solve the plate as a plane frame in the two perpendicular directions while the proposed model deals with the plate as a two dimensional problem.

It was found also that the difference between the results obtained by the codes and those obtained by the proposed model decreases with the reduction of the spring stiffness value. Based on the results shown in Figure 6, it is proposed that the column strip negative moments obtained by the ESS-95 could be increased by a factor of about 1.20 for plates with openings.

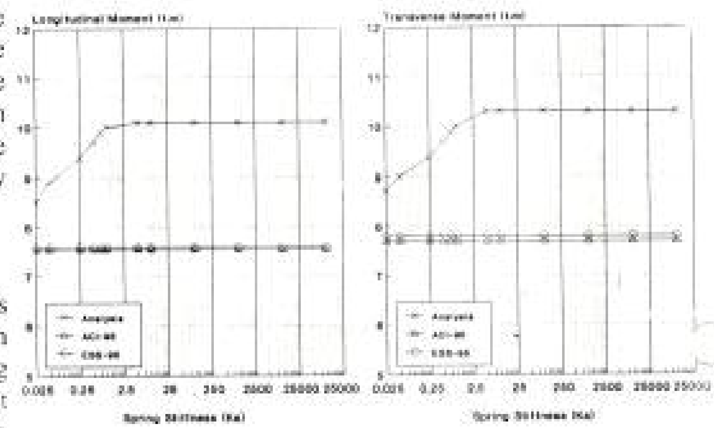
4.4 Effect of Edge Beam Size

The resulting moments for the studied plate cases with different beam sizes have been compared with those obtained from the ESS-95 and ACI-95 building codes [6,7]. The ratio of the transverse moment obtained by the proposed model to that obtained by the previously mentioned codes of practice are listed in Tables (1-a) and (1-b) for both column and field strips, respectively.

It was found from Table (1-a) that increasing the beam size results in decreasing the ratio of the analytical moment to the code moment for the negative column strip moment while the positive one was slightly affected by the size of the beam. As was mentioned in Section 4.3, the positive flexural moment values obtained by the methods stated in the codes of practice (ESS-95 and ACI-95) were higher than those obtained by the proposed analytical model, while the negative ones were lower than those obtained analytically. Table (1-a) shows also that the ratios of column strip negative and positive moments obtained



a- Positive moments



b- Negative moment

Fig. 6- Effect of column-plate connection rigidity on column strip moments

Table (1-a) Variation of Column Strip Moment

		Ratio of Analysis Moments to the Code Moments							
		t_b / t_s = 1.00 *		2.25		3.00		4.50	
		ESS-95	ACI-95	ESS-95	ACI-95	ESS-95	ACI-95	ESS-95	ACI-95
M(-ve)	S-6	1.436	1.400	1.321	1.304	1.295	1.295	1.282	1.299
	S-7	1.366	1.330	1.297	1.255	1.264	1.250	1.231	1.231
	S-8	1.419	1.393	1.267	1.220	1.219	1.196	1.181	1.181
	F-6	1.442	1.461	1.390	1.354	1.351	1.368	1.299	1.370
	F-7	1.437	1.422	1.322	1.294	1.322	1.322	1.256	1.299
	F-8	1.442	1.415	1.289	1.252	1.289	1.264	1.202	1.250
	C-6	1.461	1.480	1.355	1.338	1.342	1.360	1.329	1.403
	C-7	1.453	1.437	1.322	1.294	1.278	1.278	1.256	1.284
	C-8	1.452	1.431	1.289	1.252	1.252	1.229	1.214	1.225
M(+ve)	S-6	0.824	0.763	0.824	0.745	0.826	0.766	0.826	0.683
	S-7	0.762	0.647	0.761	0.693	0.762	0.715	0.764	0.688
	S-8	0.708	0.629	0.707	0.628	0.710	0.642	0.712	0.617
	F-6	0.840	0.778	0.837	0.775	0.842	0.797	0.833	0.750
	F-7	0.772	0.699	0.767	0.695	0.767	0.702	0.767	0.674
	F-8	0.720	0.632	0.708	0.621	0.715	0.632	0.700	0.604
	C-6	0.635	0.589	0.637	0.581	0.640	0.595	0.644	0.578
	C-7	0.563	0.512	0.565	0.515	0.573	0.538	0.581	0.522
	C-8	0.512	0.455	0.521	0.462	0.528	0.463	0.521	0.452

Table (1-b) Variation of Field Strip Moment

		Ratio of Analysis Moments to the Code Moments							
		t_b / t_s = 1.00 *		2.25		3.00		4.50	
		ESS-95	ACI-95	ESS-95	ACI-95	ESS-95	ACI-95	ESS-95	ACI-95
M(-ve)	S-6	0.615	0.593	0.577	0.577	0.577	0.577	0.577	0.577
	S-7	0.783	0.766	0.739	0.739	0.739	0.739	0.739	0.739
	S-8	0.905	0.884	0.857	0.857	0.857	0.905	0.857	0.857
	F-6	0.625	0.628	0.577	0.577	0.577	0.577	0.577	0.577
	F-7	0.739	0.739	0.739	0.739	0.739	0.739	0.739	0.739
	F-8	0.905	0.905	0.857	0.857	0.857	0.857	0.857	0.857
	C-6	0.640	0.640	0.600	0.600	0.600	0.600	0.600	0.600
	C-7	0.804	0.783	0.739	0.739	0.739	0.739	0.739	0.739
	C-8	0.918	0.905	0.905	0.905	0.905	0.905	0.905	0.905
M(+ve)	S-6	1.006	1.159	0.965	1.071	0.965	1.111	0.965	1.000
	S-7	1.225	1.319	1.227	1.377	1.259	1.447	1.214	1.308
	S-8	1.445	1.575	1.441	1.570	1.520	1.689	1.385	1.500
	F-6	0.685	0.778	0.676	0.767	0.612	0.794	0.682	0.750
	F-7	0.949	1.038	0.930	1.031	0.943	1.048	0.930	1.000
	F-8	1.161	1.286	1.142	1.264	1.129	1.296	1.129	1.207
	C-6	1.142	1.280	1.071	1.200	1.071	1.200	1.071	1.154
	C-7	1.378	1.522	1.360	1.478	1.360	1.478	1.360	1.417
	C-8	1.558	1.700	1.583	1.727	1.583	1.727	1.500	1.500

* results from reference [4].

by the proposed numerical technique to those obtained by the equivalent frame method of the ESS-95 are slightly higher than the ratios of the former moments to those obtained by the ACI-95. This was valid for the case of flat plate without edge beam ($t_b / t_s = 1.0$) and as the beam size increases the difference between the two codes decreases. Table (1-b) shows the same behavior for the field strip negative moments, however , the difference between the two codes vanishes for plates with edge beams. The opposite is true for the field strip positive moments where the ratios of the moments obtained by the proposed numerical technique to those obtained by the ACI-95 are slightly higher than the ratios of the former moments to those obtained by the equivalent frame method of the ESS-95.

In general, it is noticed that the effect of the edge beam was not significant, especially in the ESS-95 code This may be attributed to the fact that the beam size effect is not included in the equivalent frame method.

5. CONCLUSION

An analytical model was developed for studying the behavior of reinforced concrete flat plates with openings. The effect of openings arrangement, column-plate connection rigidity and edge beam size was discussed. The effectiveness of diagonal reinforcement around openings was also examined. Within the limits of the studied parameters, the following conclusions and recommendations can be drawn:

1- The proposed analytical model proved to be powerful and adequate for the analysis of reinforced concrete flat plates since it includes a high precision thin, thick layered elements for modelling plates with edge beams. Column-plate connection was simulated by a group of springs, and reinforcement around openings was easily modelled as an anisotropic smeared layer.

2- It was found that openings should not be located at a distance from the column face less than or equal to one tenth of the span. This almost agrees with the ESS-95 which prevents any opening in the area of the column head (0.25 the average span).

3- Where openings are to be placed at the intersection between two column strips, their size shall not exceed one eighth of the strip width.

4- Strengthening the openings corners located in the intersection area between two column strips by diagonal reinforcement of about 0.15 the main column strip steel results in a better redistribution of the flexural moment of the column strip.

5- It is proposed that the column strip negative moments obtained from the ESS-95 should be increased by a factor of about 1.20 for plates with openings located in column strips.

6- The effect of the edge beam was found to be insignificant, especially in the ESS-95 code, therefore it is recommended to include the effect of edge beam size in the coefficients of the equivalent frame method for distributing the moments between the column and field strips.

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